Index No. | |

Medium | **ENGLISH** |

SRI LANKAN MATHEMATICS COMPETITION 11 - 2019

September 28, 2019
10:30 am – 12 noon

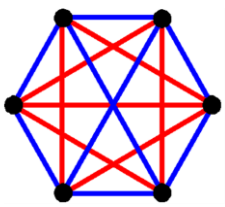
This question paper has **30 multiple choice questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

Scoring System for the Sri Lankan Mathematics Competition 11

Questions 1 to 10: 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20: 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30: 8 points for correct response, 3 points for no response, and 0 points for incorrect response.



Sri Lanka Olympiad Mathematics Foundation

1. A box has a certain number of mangoes. Sarath, Adul, Meena, Kamala and Susan guessed that the numbers of mangoes is 33, 32, 30, 28 and 27 respectively. If only one guess is correct and two guesses differ by 2 from the actual number and two guesses differ by 3 from the actual number, then who guessed correctly?

   (A) Sarath (B) Abdul (C) Meena (D) Kamala (E) Susan

2. What is the units digit of 9^{2019} ?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 9

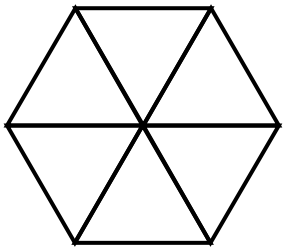
3. How many digits does the number $(111\ 111\ 111)^2$ have?

   (A) 9 (B) 16 (C) 17 (D) 18 (E) 20

4. The product $100 \times 101 \times 102$ is not divisible by

   (A) 2 (B) 3 (C) 5 (D) 17 (E) 103

5. The figure given below contains 6 equilateral triangles. How many rhombuses does it contain?



   (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

6. How many different pairs of integers (a,b) satisfy $a^b = 81$?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. The product of 2019 integers is equal to 1. Their sum cannot be equal to

   (A) -9 (B) -1 (C) 2 (D) 3 (E) 7

8. Martians use a positional number system with a base different from 10. A correct addition problem in their system is shown below using our Hindu-Arabic numerals:

$$\begin{array}{r} 4421 \\ +\ 1154 \\ \hline 5605 \end{array}$$

What is their base?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

9. How many different positive divisors does 72 have?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

10. In how many ways can a path starting from the box containing S and ending in a box containing the number 11 be traversed through boxes containing S, L, M, C, and 11 in that order if the path consists of horizontal, vertical and/or diagonally down straight line segments connecting a pair of neighboring (a box can have at most 8 neighbors above, below, left, right and diagonal) boxes?

S	L	M	C	11
L	L	M	C	11
M	M	M	C	11
C	C	C	C	11
11	11	11	11	11

(A) 45 (B) 48 (C) 50 (D) 52 (E) 69

11. Sarath, Abdul, Kamal, Sanjeeva, and Anwar participated in a bicycle race at a Sinhala and Tamil New Year Festival and Hannah, Kamala, Meena and Susan said the following before the race:
Hannah: Sarath or Abdul will win.
Kamala: Sanjeeva or Anwar will win.
Meena: Sarath or Sanjeeva will win.
Susan: Sanjeeva or Abdul will win.
Only one of them was right. Who won the race?

(A) Sarath (B) Abdul (C) Kamal (D) Sanjeeva (E) Anwar

12. Little Nimal has to distribute 4 hats colored red, blue, green and yellow to his 4 friends Kamal, Sanjeeva, Anwar and Hannah whose favorite colors are red, blue, green and yellow respectively. In how many ways can Nimal distribute hats so that only one gets his or her favorite colored hat?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 8

13. Consider the following statements:
    All mathematicians like the color red.
    No politician is logical.
    Illogical people do not like the color red.
    Which of the following is/are valid conclusions?
    I. Politicians do not like the color red.
    II. Some politicians like the color red.
    III. Illogical people are not mathematicians.

    (A) I only (B) II only (C) III only (D) I and III only (E) II and III only

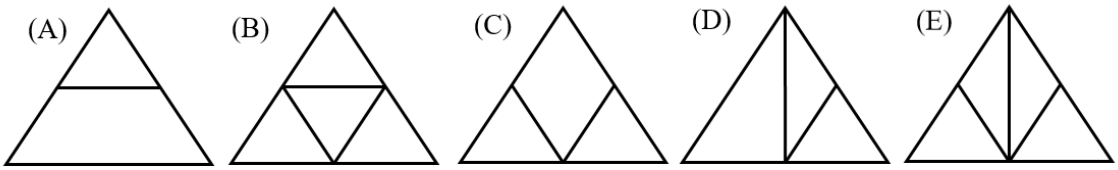
14. For positive real numbers a and b, define $a \otimes b = \dfrac{a \times b}{a + b}$ where $\times$ and $+$ are ordinary multiplication and addition respectively. How many different pairs of integers (a, b) satisfy $a \otimes b = 1$?

    (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. A supermarket has 50 boxes of mangoes. Each box contains between 50 to 60 mangoes inclusive of 50 and 60. What is the largest integer n such that there are at least n boxes containing the same number of mangoes?

    (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

16. Which figure cannot be drawn without lifting the pencil and without going over a straight line segment again?



17. Sarath and Meena play a game in which they take turns in adding a positive integer less than the current number to the current number. They start with 2. The player who reaches 100 first wins the game. Which of the following is/are true?

    I. The player who plays first (first player) has a winning strategy.
    II. First player can always win in the 5^{th} move.
    III. The player who plays second (second player) has a winning strategy if the game starts with 3.

    (A) I only (B) II only (C) III only (D) I and III only (E) All

18. *A* is the collection of positive integers that are squares and multiples of 12. Which of the following is/are true?

I. *A* has infinitely many numbers.
II. *A* has a number whose sum of digits is 9.
III. *A* has a number whose sum of digits is 18.

(A) I only (B) II only (C) III only (D) II and III only (E) All

19. If a sequence of numbers $a_1, a_2, a_3, \ldots$ is given by $a_{n+1} = \frac{1}{2-a_n}$ for $n > 1$ and $a_1 = \frac{1}{4}$, then the value of a_{2019} is

(A) $\frac{1}{4}$ (B) $\frac{4}{7}$ (C) $\frac{3}{4}$ (D) $\frac{6055}{6058}$ (E) $\frac{6058}{6061}$

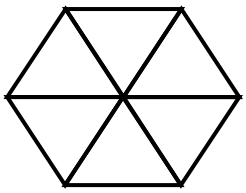
20. In the following addition problem, different letters take different digits but none of them takes 8.

$$\begin{array}{r} SLMC8 \\ +\quad IS \\ \hline SUPER \end{array}$$

What is the sum of the digits of the maximum number *SUPER* can take?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

21. A regular hexagon of side length 1 can be made by using 6 equilateral triangles of side length 1 as follows:



How many equilateral triangles of side length 1 are needed to make a regular hexagon of side length 3?

(A) 48 (B) 54 (C) 60 (D) 62 (E) 64

22. Consider the binary operation on positive real numbers given in question 14: For positive real numbers *a* and *b*, define $a \otimes b = \frac{a \times b}{a + b}$ where × and + are ordinary multiplication and addition respectively. Which of the following is/are true?

I. For all positive real numbers *a* and *b*, $a \otimes b = b \otimes a$.
II. For all positive real numbers *a*, *b* and *c*, $(a \otimes b) \otimes c = a \otimes (b \otimes c)$.

III. For all positive real numbers *a*, *b* and *c*, $a \otimes (b+c) = a \otimes b + a \otimes c$.

(A) I only (B) II only (C) III only (D) I and II only (E) All

23. How many positive integer solutions does $15x + 6y = 2019$ have?

(A) 67 (B) 68 (C) 134 (D) 135 (E) 2019

24. A fair die is rolled 3 times. What is the probability of getting at most a 4 at least 2 times?

(A) $\frac{8}{27}$ (B) $\frac{12}{27}$ (C) $\frac{13}{27}$ (D) $\frac{20}{27}$ (E) $\frac{21}{27}$

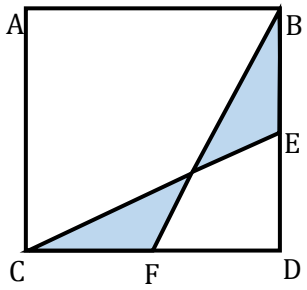
25. The smallest integer larger than $(\sqrt{2019} + \sqrt{2020})^2$ is

(A) 8077 (B) 8078 (C) 8079 (D) 8080 (E) 8081

26. The number $\frac{\sqrt{3+\sqrt{5}}}{\sqrt{2}+\sqrt{10}}$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{\sqrt{2}}$ (E) $\frac{\sqrt{5}}{\sqrt{2}}$

27. *ABCD* is a square and *E* and *F* are the midpoints of *BD* and *CD* respectively. What is the ratio, area of shaded region to area of unshaded region?



(A) 1 : 3 (B) 1 : 4 (C) 1 : 5 (D) 1 : 6 (E) 4 : 7

28. When a positive integer d divides each of the numbers 655, 1156 and 1490, the remainder is the same positive integer r. What is $d - r$?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

29. In a tennis tournament the ratio of men to women who participated was 3 : 1 and each player played exactly once with every other player. None of the matches ended in a tie and the ratio of the number of games men won to the number of games women won was 3 : 2. How many participated in this tournament if it was a number between 42 and 58?

(A) 44 (B) 48 (C) 52 (D) 56 (E) 57

30. In the following addition problem, different letters take different digits but none of them takes 1.

$$\begin{array}{r} SLMC \\ \quad 11 \\ +\quad IS \\ \hline NEAT \end{array}$$

What is the sum of the digits of the maximum number *NEAT* can take?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

Thank you very much for your participation in the Sri Lankan Mathematics Competition SLMC 13 - 2019. Your score on this competition will be posted against your index number in www.slmathsolympiad.org. In this competition we have tried to showcase mathematics by posing puzzle type problems covering various areas of mathematics. Though the problems require very little knowledge of mathematics, not more than a Year 6 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your interest in mathematics beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! You have to learn problem solving strategies. Solve math problems for fun. Doing mathematics outside the school curriculum box will greatly improve your school mathematics.

As you know doing these problems in the exam hall under the pressure of time is difficult. This way may not bring the best in you. We hope that you will leisurely do and think about these problems after the competition. Looking back at the problems you solved and reflecting on them will improve your mathematical thinking. Some of these problems have deep mathematical ideas in them. History shows us that some mathematical ideas we have to learn in school evolved through long periods of time baffling the greatest mathematical minds in those times. For example negative numbers. Leo Rogers says at http://nrich.maths.org/5961:

"Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician Brahmagupta, it is surprising that in 1758 the British mathematician Francis Maseres was claiming that negative numbers "... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple".

Read that article. Mathematics is a beautiful subject. But to see the beauty you have to engage in good mathematics. We hope that this competition will help you to see the beauty in mathematics.

For any comments/suggestions: info@slmathsolympiad.org