Index No.

Medium **ENGLISH**

SRI LANKAN MATHEMATICS COMPETITION 13 - 2019

September 28, 2019
10:30 am – 12 noon

This question paper has **30 multiple choice questions**. The duration of this competition is **90 minutes**. **Answer all questions**. Please read the questions carefully and **fill in the correct lettered circle (only one) against the correct question number in the given answer sheet**. Note that no responses get at least two points while incorrect responses receive zero points. **Please write your index number in the box provided at the top right corner of your question paper**.

Scoring System for the Sri Lankan Mathematics Competition 13

Questions 1 to 10: 5 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 11 to 20: 6 points for correct response, 2 points for no response, and 0 points for incorrect response.

Questions 21 to 30: 8 points for correct response, 3 points for no response, and 0 points for incorrect response.

Sri Lanka Olympiad Mathematics Foundation

1. A supermarket has 50 boxes of mangoes. Each box contains between 50 to 60 mangoes inclusive of 50 and 60. What is the largest integer n such that there are at least n boxes containing the same number of mangoes?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

2. Sarath, Abdul, Kamal, Sanjeeva, and Anwar participated in a bicycle race at a Sinhala and Tamil New Year Festival and Hannah, Kamala, Meena and Susan said the following before the race:
   Hannah: Sarath or Abdul will win.
   Kamala: Sanjeeva or Anwar will win.
   Meena: Sarath or Sanjeeva will win.
   Susan: Sanjeeva or Abdul will win.
   Only one of them was right. Who won the race?

   (A) Sarath (B) Abdul (C) Kamal (D) Sanjeeva (E) Anwar

3. Little Nimal has to distribute 4 hats colored red, blue, green and yellow to his 4 friends Kamal, Sanjeeva, Anwar and Hannah whose favorite colors are red, blue, green and yellow respectively. In how many ways can Nimal distribute hats so that only one gets his or her favorite colored hat?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 8

4. Consider the following statements:
   All mathematicians like the color red.
   No politician is logical.
   Illogical people do not like the color red.
   Which of the following is/are valid conclusions?
   I. Politicians do not like the color red.
   II. Some politicians like the color red.
   III. Illogical people are not mathematicians.

   (A) I only (B) II only (C) III only (D) I and III only (E) II and III only

5. In how many ways can a path starting from the box containing S and ending in a box containing the number 13 be traversed through a total of 5 boxes containing S, L, M, C, and 13 in that order if the path consists of horizontal, vertical and/or diagonally down straight line segments connecting a pair of neighboring (a box can have at most 8 neighbors above, below, left, right and diagonal) boxes?

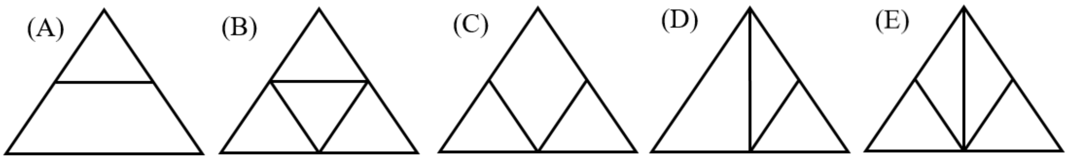
S	L	M	C	13
L	L	M	C	13
M	M	M	C	13
C	C	C	C	13
13	13	13	13	13

(A) 45 (B) 48 (C) 50 (D) 52 (E) 69

6. For positive real numbers a and b, define $a \otimes b = \frac{a \times b}{a + b}$ where $\times$ and $+$ are ordinary multiplication and addition respectively. How many different pairs of integers (a, b) satisfy $a \otimes b = 1$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. Which figure cannot be drawn without lifting the pencil and without going over a straight line segment again?



8. Sarath and Meena play a game in which they take turns in adding a positive integer less than the current number to the current number. They start with 2. The player who reaches 100 first wins the game. Which of the following is/are true?

I. The player who plays first (first player) has a winning strategy.
II. First player can always win in the 5^{th} move.
III. The player who plays second (second player) has a winning strategy if the game starts with 3.

(A) I only (B) II only (C) III only (D) I and III only (E) All

9. *A* is the collection of positive integers that are squares and multiples of 12. Which of the following is/are true?

I. *A* has infinitely many numbers.
II. *A* has a number whose sum of digits is 9.
III. *A* has a number whose sum of digits is 18.

(A) I only (B) II only (C) III only (D) II and III only (E) All

10. In the following addition problem, different letters take different digits but none of them takes 8.

$$\begin{array}{r} S\,L\,M\,C\,8 \\ +\quad\quad I\,S \\ \hline S\,U\,P\,E\,R \\ \hline \end{array}$$

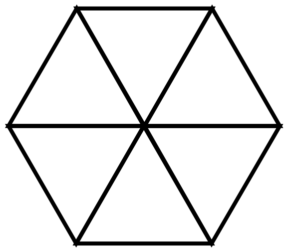
What is the sum of the digits of the maximum number *SUPER* can take?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

11. If a sequence of numbers $a_1, a_2, a_3, \ldots$ is given by $a_{n+1} = \frac{1}{2-a_n}$ for $n \geq 1$ and $a_1 = \frac{1}{4}$, then the value of a_{2019} is

(A) $\frac{1}{4}$ (B) $\frac{4}{7}$ (C) $\frac{3}{4}$ (D) $\frac{6055}{6058}$ (E) $\frac{6058}{6061}$

12. A regular hexagon of side length 1 can be made by using 6 equilateral triangles of side length 1 as follows:



How many equilateral triangles of side length 1 are needed to make a regular hexagon of side length 3?

(A) 48 (B) 54 (C) 60 (D) 62 (E) 64

13. Consider the binary operation on positive real numbers given in question 6: For positive real numbers *a* and *b*, define $a \otimes b = \dfrac{a \times b}{a+b}$ where $\times$ and $+$ are ordinary multiplication and addition respectively. Which of the following is/are true?

I. For all positive real numbers *a* and *b*, $a \otimes b = b \otimes a$.

II. For all positive real numbers *a*, *b* and *c*, $(a \otimes b) \otimes c = a \otimes (b \otimes c)$.

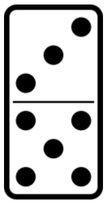
III. For all positive real numbers *a*, *b* and *c*, $a \otimes (b+c) = a \otimes b + a \otimes c$.

(A) I only (B) II only (C) III only (D) I and II only (E) All

14. How many positive integer solutions does $15x + 6y = 2019$ have?

(A) 67 (B) 68 (C) 134 (D) 135 (E) 2019

15. A domino is a thin rectangular piece of wood whose top face is divided into two squares. Each of the squares is either blank or contains between one and six dots inclusive of one and six. See the image below:



If in a domino each of the squares is either blank or contains between one and nine dots inclusive of one and nine and the patterns of the dots is irrelevant, then how many different dominoes are there ?

(A) 50 (B) 55 (C) 60 (D) 65 (E) 70

16. A fair die is rolled 3 times. What is the probability of getting at most a 4 at least 2 times?

(A) $\frac{8}{27}$ (B) $\frac{12}{27}$ (C) $\frac{13}{27}$ (D) $\frac{20}{27}$ (E) $\frac{21}{27}$

17. Suppose that the statement "all Sri Lankans are liars or gullible" is made by a Sri Lankan and a liar always lies. Which of the following statements is/are true?

I. If the statement is true then the person who made the statement is gullible.
II. If the statement is true then the person who made the statement is not gullible.
III. If the statement is false then there is at least one person who is not a liar and not gullible.

(A) I only (B) II only (C) III only
(D) I and III only (E) II and III only

18. When a positive integer d divides each of the numbers 655, 1156 and 1490, the remainder is the same positive integer r. What is $d-r$?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

19. The smallest integer larger than $(\sqrt{2019}+\sqrt{2020})^2$ is

(A) 8077 (B) 8078 (C) 8079 (D) 8080 (E) 8081

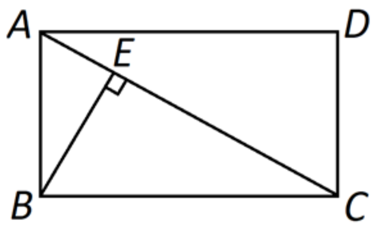
20. In the following addition problem, different letters take different digits but none of them takes 1.

$$\begin{array}{r} S\ L\ M\ C \\ 1\ 1 \\ +\quad I\ S \\ \hline N\ E\ A\ T \end{array}$$

What is the sum of the digits of the maximum number *NEAT* can take?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

21. *ABCD* is a rectangle where $AB = 15$, $AD = 20$ and *BE* is perpendicular to *AC*. What is the ratio, area of *ABE*: area of *ABCD*?



(A) 9 : 16 (B) 9 : 25 (C) 12 : 25 (D) 16 : 25 (E) 9 : 50

22. In a tennis tournament the ratio of men to women who participated was 3 : 1 and each player played exactly once with every other player. None of the matches ended in a tie and the ratio of the number of games men won to the number of games women won was 3 : 2. How many participated in this tournament if it was a number between 42 and 58?

(A) 44 (B) 48 (C) 52 (D) 56 (E) 57

23. A sequence is obtained from the positive integer sequence by removing perfect squares and perfect cubes. What is the 2019$^{\text{th}}$ term of this sequence if the numbers in the sequence are in the ascending order?

(A) 2070 (B) 2071 (C) 2072 (D) 2073 (E) 2074

24. The number $\dfrac{\sqrt{3+\sqrt{5}}}{\sqrt{2}+\sqrt{10}}$ is equal to

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{\sqrt{2}}$ (D) $\dfrac{\sqrt{3}}{\sqrt{2}}$ (E) $\dfrac{\sqrt{5}}{\sqrt{2}}$

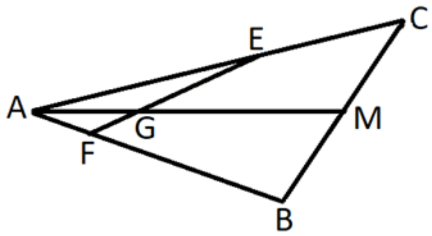
25. $f(n)$ is a unique positive integer for each positive integer n such that,
(a) $f(n + 2019) \geq f(n) + 2019$ for each positive integer n,
(b) $f(n + 1) \leq f(n) + 1$ for each positive integer n and
(c) $f(1) = 10$.
What is the value of $f(2019)$?

(A) 2019 (B) 2020 (C) 2021 (D) 2027 (E) 2028

26. Positive integers a, b and c satisfy $13a^2 - 3ab + 7c^2 = 0$. Then which of the following is/are true? (Note: $\gcd(a, b)$ means the greatest common divisor of a and b.)
I. 3 divides $\gcd(a, b)$.
II. 3 divides $\gcd(a, c)$.
III. $\gcd(a, b) . \gcd(b, c) . \gcd(c, a)$ can be equal to 3.

(A) I only (B) II only (C) III only
(D) I and III only (E) II and III only

27. In the triangle *ABC* shown in the figure below, *M* is the mid-point of side *BC*, *AB* = 6 and *AC* = 8.



If *AE* : *AF* = 4 : 1, then what is *EG* : *GF*?

(A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 3 : 2 (E) 3 : 4

28. In a round robin (each player plays exactly one game against every other player) chess tournament ten players took part. In this tournament a win, a draw and a loss were worth 1, 0, -1 points respectively. If all the players got different scores at the end of the tournament what percentage of the games did not end in a draw?

(A) Less than 5%
(B) Between 6% and 10%
(C) Between 11% and 15%
(D) Between 16% and 29%
(E) More than 30%

29. The working committee of the *Tea Party* in the *Land of Lairs* consists of 3 subcommittees, *A*, *B* and *C*. A member of the working committee belongs to one and only one subcommittee. During election time the subcommittees undergo the operation of removing one each from two subcommittees and adding one to the remaining subcommittee. Let x_n, y_n and z_n denote the number of members in *A*, *B* and *C* respectively after the *n*th operation for $n \geq 0$ where $n = 0$ corresponds to the subcommittees before the operations begin. Which of the following statements on the changing sizes of the subcommittees under this operation is/are false?

I. If $(x_0, y_0, z_0) = (45, 24, 30)$, then $x_n = z_n$ for some *n*.
II. If $(x_0, y_0, z_0) = (49, 29, 21)$, then $x_n = y_n = z_n = 21$ for some *n*.
III. If $(x_0, y_0, z_0) = (49, 29, 21)$, then $(x_n, y_n, z_n) = (43, 33, 19)$ for some *n*.

(A) I only (B) I and II only (C) I and III only
(D) II and III only (E) All

30. Let a_n be the number of strings of length *n* of English uppercase letters that contain SLMC. For example SLMCISGREAT is a string of length 11 that contains SLMC and SRILANKANMATHEMATICSCOMPETITION is a string of length 31 that does not contain SLMC. Clearly $a_1 = a_2 = a_3 = 0$. Which of the following is/are true?

I. $a_6 = 3 \times 26^2$.
II. $a_n = 26a_{n-1} + 26^{n-4}$ for $n \geq 5$.
III. $a_n = 26a_{n-1} - a_{n-4} + 26^{n-4}$ for $n \geq 5$.

(A) I only (B) II only (C) III only
(D) I and II only (E) I and III only

Thank you very much for your participation in the Sri Lankan Mathematics Competition SLMC 13 - 2019. Your score on this competition will be posted against your index number in www.slmathsolympiad.org. In this competition we have tried to showcase mathematics by posing puzzle type problems covering various areas of mathematics. Though the problems require very little knowledge of mathematics, not more than a Year 6 student's basic mathematics knowledge, some problems might require the mathematical maturity of a student in a higher grade. We hope that this kind of problems will stimulate your interest in mathematics beyond classroom mathematics. If you didn't do too well, don't be discouraged! You may have great mathematical talent, but it requires nurturing!! You have to learn problem solving strategies. Solve math problems for fun. Doing mathematics outside the school curriculum box will greatly improve your school mathematics.

As you know doing these problems in the exam hall under the pressure of time is difficult. This way may not bring the best in you. We hope that you will leisurely do and think about these problems after the competition. Looking back at the problems you solved and reflecting on them will improve your mathematical thinking. Some of these problems have deep mathematical ideas in them. History shows us that some mathematical ideas we have to learn in school evolved through long periods of time baffling the greatest mathematical minds in those times. For example negative numbers. Leo Rogers says at http://nrich.maths.org/5961:

"Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician Brahmagupta, it is surprising that in 1758 the British mathematician Francis Maseres was claiming that negative numbers "... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple".

Read that article. Mathematics is a beautiful subject. But to see the beauty you have to engage in good mathematics. We hope that this competition will help you to see the beauty in mathematics.

For any comments/suggestions: info@slmathsolympiad.org